NOTES ON FIXED VERSUS RANDOM EFFECTS

Basic Model

$$y_{ij} = \alpha_i + X_{ij}\beta + \varepsilon_{ij}$$

where y_{ij} is observation j from group i, X_{ij} is a vector of non-random explanatory variables, β is a vector of unobserved parameters of theoretical interest, and ε_{ij} is a random error. The group may consist of observations on a particular unit (i.e., country or survey respondent) over time, in which case we recognize the data set as a panel, or multiple observations from the same sampling unit (i.e., students in a classroom or survey respondents from a congressional district), in which case we recognize the data as a clustered sample. The former will typically involve issues of autocorrelation of errors; the latter correlation among errors from a particular sampling unit.

Fixed-Effects (or Least Squares Dummy Variable Model)

Assumption: $\alpha_i \neq \alpha$

Motivation: Unchanging characteristics of the group are related to the dependent variable — the French are French, the Old South is different, some teacher can teach anything better. If these factors are correlated with any of the explanatory variables, then excluding them will result in biased OLS estimates of β. Their average effect can be taken into account by including indicator variables for each group. This is typically done in the standard way of including an overall constant and indicators for i-1 of the groups.

Note: In panels, it may be appropriate also to include fixed effects for time periods if it is possible that there are contemporaneous effects on all units, such as world economic conditions or technological change in the case of country panels, or salient events in the case of respondents.

Advantages:

1. Does not require assumption that group differences are uncorrelated with explanatory variables.

2. Ease of estimation and interpretation — just basic OLS model with large number of indicators. (Though in practice typically implemented with group averages.)

Disadvantages:

1. Does not have natural interpretation of generalization to larger population. Inferences are conditional on the particular set of groups in estimation.

2. Fixed effects likely to be imprecise when number of units within groups is small.

3. Estimating large number of parameters uses up degrees of freedom.

4. Model cannot include both fixed effects and unit-invariant variables. Variables that do not vary much across units within the group cannot be precisely estimated.

Random Effects (Error Components Model)

Assumption: $\alpha_i = \alpha + \eta_i \quad \eta_i \sim \mathbf{N}[0, \sigma_\eta^2]$

$$y_{ij} = \alpha + X_{ij}\beta + u_{ij} \quad u_{ij} = \varepsilon_{ij} + \eta_i$$

The group effect can be thought of as a random deviation from some common mean. A consequence of η_i is a correlation among units within a group equal to $\sigma_\eta^2 / (\sigma_\eta^2 + \sigma_\varepsilon^2)$, thus estimation usually involves either Feasible GLS or MLE.

Advantages:

1. Natural interpretation of groups as random draw from distribution and estimates of β are not conditional on the particular groups included.

2. Allows for the inclusion of explanatory variables that do not vary within groups, important if theories are about these invariant effects.

3. If group variance component is the correct specification, then random effects estimation will be more efficient than OLS with fixed effects. (Subject to usual caveat of difference between GLS and FGLS.)

Disadvantage (big):

Assumes that η_i is uncorrelated with the variables in X_{ij}. If assumption is violated, then errors will be correlated with the independent variables and the estimator of β will be biased and inconsistent.

A Hausman Test can be constructed to check for independence. Under null, both random and fixed effects estimators of β should be similar, so reject if very different. Problem: test can't be performed when random effects are being used specifically because fixed effects are not feasible.

Let b_F and b_R be the vectors of estimates except for the constant in the fixed and random effects models, respectively. Let Σ be the difference between the respective estimated variance-covariance matrices for b_F and b_R. (Under the null, the fixed effects model is inefficient, so it will have a "larger" variance covariance matrix.) Then the following is distributed asymptotically χ^2 with degrees of freedom equal to the number of independent variables:

$$[b_F - b_R]' \Sigma^{-1} [b_F - b_R]$$

Generalization: Hierarchical Models (Random or Mixed Coefficient Models)

Construct a model in which some or all of the elements of β are treated as a function of group characteristics and a random error.

The random effects model is a special case in which all the elements of β are treated as non-random and independent of group characteristics and the constant is treated as random.

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